References

[1]Bryson, A. E., and Ho, Y.-C., *Applied Optimal Control,* Hemisphere, New York, 1975.

[2]Norbutas, R. J., *Optimal Interception Guidance for Multiple Target Set,* Electronics Systems Lab., Massachusetts Inst. of Technology, AD-675975, ESL-R-333, Cambridge, MA, Jan. 1968.

[3]King, J. T., *Introduction to Numerical Analysis*, McGraw-Hill, New York, 1984.

[4]Wolfram, S., *Mathematica,* Addison-Wesley, Reading, MA, 1988.

Gain Scheduling Simplification by Simultaneous Control Design

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I. Introduction

THE robust control problem can be stated as follows: Design a single controller $C(s)$ that achieves prescribed performance over a region of operating conditions of the plant $G(s)$. This performance can be achieved by designing a controller for a representative plant $G_0(s)$ and maximizing the perturbation, around $G_0(s)$, that can be tolerated by the controller $C(s)$. The goal of the maximization is to cover the entire region of operation. This problem has been addressed in Refs. 1-3. Another version of this problem is to look at a "discrete" representation of the region of operation. In this approach, a finite number, say, M, of representative plants are chosen. A controller $C(s)$ is then designed to control all of the plants.[4-12] We will refer to this problem as the *simultaneous control problem.*

One of the motivations behind the simultaneous control problem is to control a nonlinear system, represented by linear models at different operating points, using one controller. In this case, each linear model represents an operating point. The parameter variations of the system model form a low-frequency upper bound on the singular values of the loop-gain transfer function. Some of the robust controller design methods, LQG/LTR, for example, have no mechanism for dealing with this upper bound. On the other hand, simultaneous control is a natural way of approaching this issue. The simultaneous control technique can make gain scheduling easier to implement by reducing the number of operating points to be scheduled. This is accomplished by grouping the total number of prespecified operating points into classes. A different controller is then designed for each of these classes.

Another use for simultaneous control is in the design of controllers that are robust against sensor or actuator failures. When an actuator or a sensor fails during operation, the system characteristics change, effectively generating a new system. Simultaneous control design can be used to design a single controller that gives good performance for both the original system and the new system generated by the failure.

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The simultaneous control problem has attracted many researchers over the last few years, but most of their work has dealt with the problem of simultaneous stabilization without meeting any specified performance objectives. Some of the results for single-input/single-output (SISO) transfer functions were reported in Refs. 4–8. The multi-input/output case was addressed in Ref. 9. In that work, a dynamical output-feedback controller is designed to achieve the stabilization of more than one system. A nonlinear feedback controller for SISO systems was used in Ref. 11. A state-feedback controller is proposed in Ref. 12. The design in Ref. 12 is a two-stage design. In one stage a state-feedback gain K is designed to make all of the systems minimum-phase and the pole-zero excess of each system equal to 1. Once this is achieved, a constant output-feedback controller can be designed to stabilize all of the "new" systems.

The only work that deals with the problem of simultaneous attainment of performance objectives is that of Looze.[10] The objective there is to find a state-feedback controller K that minimizes an objective function composed of the sum of M standard LQR cost functions. Each one of these cost functions penalizes the states and controls of one of the M systems. One problem with the work in Ref. 10 is the need for finding an initial stabilizing controller. A method for generating such a controller was given, but it requires a lot of CPU time.

In this paper we address the problem of designing a state-variable feedback controller that stabilizes a finite collection of systems. Our technique has extra design parameters that allow, in addition to stabilization, the achievement of some performance specifications. An initial stabilizing gain is not needed for our technique. We design a controller K_i for each of the M systems S_i and give necessary conditions for the sum of K_i to be a stabilizing controller for all of the systems S_i, $i = 1, \ldots, M$.

The main result is presented in Sec. II, followed by a gain-scheduling design example. In the design example we consider the problem of stabilizing the longitudinal short-period approximation of the McDonnell-Douglas F4-E fighter aircraft for different operating points.

II. Simultaneous Control Results and Gain-Scheduling Example

We consider a collection of linear time-invariant systems described by

$$\dot{x}_i = A_i x_i + B_i u_i, \qquad i = 1, 2, \ldots, M \tag{1}$$

We assume that (A_i, B_i) is controllable for all i and we seek a controller K that stabilizes the collection of M systems. The following theorem shows one how to construct such a controller.

Theorem

A state-variable feedback controller $u = -Kx$ that stabilizes all of the systems in Eq. (1) exists if there exist Q_i and positive definite R_i such that

$$Q_i + P_i B_i R_i^{-1} B_i^t P_i + 2 P_i B_i \left(\sum_{i \neq j} R_i^{-1} B_i^t P_i \right) > 0 \tag{2}$$

where P_i is the solution of the algebraic Riccati equation

$$A_i^t P_i + P_i A_i - P_i B_i R_i^{-1} B_i^t P_i + Q_i = 0 \tag{3}$$

Moreover, the controller that stabilizes the M systems is given by

$$K = -\sum_{i=1}^{M} R_i^{-1} B_i^t P_i \tag{4}$$

Remark: Note that Q_i need not be positive semidefinite as is usually assumed. It was shown in Ref. 13 that, in the standard linear quadratic regulator, an indefinite Q is sometimes needed to achieve certain performance specifications.

Proof: Define M Lyapunov functions $V_i = x_i^t P_i x_i$. The systems given by Eq. (1) are all stable under the control gain given by Eq. (4) if the derivatives of all V_i are negative. The derivative of V_i with respect to time is given by

$$\dot{V}_i = x_i^t [A_i^t P_i + P_i A_i] x_i + 2x_i^t P_i B_i u_i \tag{5}$$

By selecting P_i as the solution of ARE in Eq. (3), V_i can be written as

$$\dot{V}_i = -x_i^t [Q_i + P_i B_i R_i^{-1} B_i^t P_i] x_i + 2x_i^t P_i B_i u_i \tag{6}$$

Now we use the controller given in Eq. (4) to obtain the result in Eq. (2). □

Note that the controller gain K_i for each system is the LQR optimal gain and is given by

$$K_i = -R_i^{-1} B_i^t P_i$$

where P_i is the solution to Eq. (3). Note further that the design equations are in terms of open-loop quantities so that no initial stabilizing gain is needed.

Design Example

We apply our results to the control of the short-period longitudinal modes of the McDonnell-Douglas F4-E aircraft.[11] This is a three-state model. The states are normal acceleration n_z, pitch rate q, and elevator angle Δ_e. We consider three operating points representing the following three flight conditions: 1) Mach 0.5, altitude 5000 ft; 2) Mach 0.9, altitude 35,000 ft; and 3) Mach 0.85, altitude 5000 ft. The system matrices corresponding to these flight conditions are

$$A_1 = \begin{bmatrix} -0.9896 & 17.41 & 96.15 \\ 0.2648 & -0.8512 & -11.39 \\ 0.0 & 0.0 & -30.0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} -97.78 \\ 0.0 \\ 30.0 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -0.6607 & 18.11 & 84.34 \\ 0.08201 & -0.6587 & -10.81 \\ 0.0 & 0.0 & -30.0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -272.2 \\ 0 \\ 30.0 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} -1.702 & 50.72 & 263.5 \\ 0.2201 & -1.412 & -31.99 \\ 0.0 & 0.0 & -30.0 \end{bmatrix}, \quad B_3 = \begin{bmatrix} -85.09 \\ 0.0 \\ 30.0 \end{bmatrix}$$

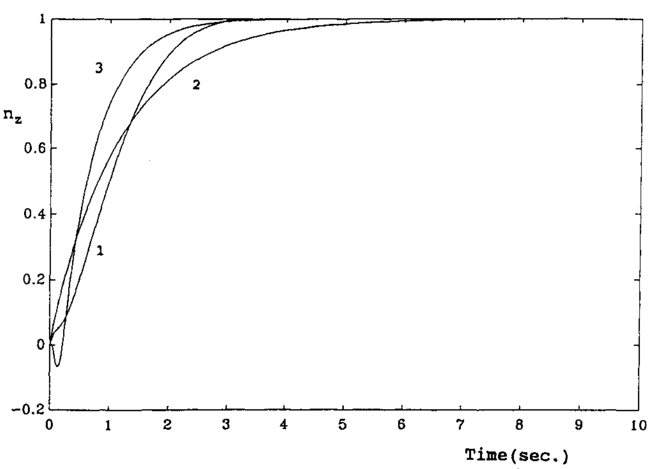

Fig. 1 Step response of the three systems.

We would like to design an acceleration command system for these three systems. The function of this command system is to hold the aircraft acceleration at a commanded or constant reference value r. To do this, an integrator is added in the acceleration channel. We now need to design a single controller for the three operating points for the purpose of gain scheduling. The state-feedback controller

$$K = [-0.7011 \quad -5.9106 \quad 0.1261 \quad 0.9487]$$

easily found using Eqs. (2–4) achieves the intended purpose. The step response of the closed-loop system with initial conditions $[1 \quad 0 \quad 0 \quad 0]^t$ is shown in Fig. 1. The controller resulted in acceptable responses for all three operating points.

Note that this state-feedback controller needs the undesirable feedback of the elevator angle. An output-feedback controller can be used to avoid this; this issue is addressed in Ref. 14.

III. Conclusions

We proposed a design technique for the simultaneous stabilization of a collection of linear systems. An optimal controller for each of the systems is designed and necessary conditions for the sum of the controllers to be stabilizing for all of the systems were given. These results were used to design a single state-feedback controller to stabilize the longitudinal short-period model of the F4-E aircraft at three different operating points.

References

[1]Zames, G., and Francis, B., "Feedback, Minimax Sensitivity, and Optimal Robustness," *IEEE Transactions on Automatic Control*, Vol. AC-28, No. 5, 1983, pp. 585–601.

[2]Moore, J., and Lige, X., "Loop Recovery and Robust State Estimate Feedback Design," *IEEE Transactions on Automatic Control*, Vol. AC-32, No. 6, 1987, pp. 512–517.

[3]Paduano, J., and Downing, D., "Sensitivity Analysis of Digital Flight Control Systems Using Singular-Value Concepts," *Journal of Guidance, Control, and Dynamics*, Vol. 12, No. 3, 1989, pp. 297–303.

[4]Saeks, R., Murray, J., Chua, O., Karmokolias, C., and Iyer, I., "Feedback Systems Design: The Single-Variate Case—Part II," *Circuits System Signal*, Vol. 44, No. 2, 1983, pp. 3–34.

[5]Debowski, A., and Kurylowicz, A., "Simultaneous Stabilization of Linear Single-Input/Single-Output Plants," *International Journal of Control*, Vol. 44, No. 5, 1986, pp. 1257–1264.

[6]Saeks, R., and Murray, J., "Fractional Representation Algebraic Geometry, and the Simultaneous Stabilization Problem," *IEEE Transactions on Automatic Control*, Vol. AC-27, Aug. 1982, pp. 895–903.

[7]Vidyasagar, M., and Viswanadham, N., "Algebraic Design Techniques for Reliable Stabilization," *IEEE Transactions on Automatic Control*, Vol. AC-27, Oct. 1982, pp. 1085–1095.

[8]Vidyasagar, M., "Some Results on Simultaneous Stabilization with Multiple Domains of Stability," *Automatica*, 1987, pp. 535–540.

[9]Ghosh, B., and Byrnes, C., "Simultaneous Stabilization and Pole-Placement by Nonswitching Dynamic Compensation," *IEEE Transactions on Automatic Control*, Vol. AC-28, No. 6, June 1983, pp. 735–741.

[10]Looze, D., "A Dual Optimization Procedure for Linear Quadratic Robust Control Problems," *Automatica*, Vol. 19, No. 3, 1983, pp. 299–302.

[11]Peterson, I., "A Procedure for Simultaneously Stabilizing a Collection of Single Input Linear Systems Using Non-Linear State Feedback Control," *Automatica*, Vol. 23, No. 1, 1987, pp. 33–40.

[12]Schmitendorf, W., and Holot, C., "Simultaneous Stabilization via Linear State Feedback Control," *IEEE Transactions on Automatic Control*, Vol. AC-34, No. 9, 1989, pp. 1001–1005.

[13]Al-Sunni, F., Stevens, B., and Lewis, F., "Negative State Weighting in the Linear Quadratic Regulator for Aircraft Control," *Journal of Guidance, Control, and Dynamics*, Vol. 15, No. 5, 1992, pp. 1279–1281.

[14]Al-Sunni, F., and Lewis, F., "Simultaneous Output-Feedback Stabilization for Continuous Systems," *IFAC Workshop in System Structure and Control*, Sept. 1992, pp. 360–363